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Control Design and Analysis of Active Vehicle Suspension using Integral Pole Placement Controller

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Abstract—*This paper presents the integral pole placement controller design for controlling the suspension travel of a quarter car active model. Apart from good ride comfort and handling another equally important suspension's objective is to support the vehicle static weight. The latter objective can be achieved by keeping the suspension travel space within the rattle space limits. For developing the controller the quarter car active suspension model is developed using the Newton's second law of motion with two degrees of freedom. The developed mathematical model of the quarter car is then modelled into MATLAB/Simulink environment. Firstly open loop simulations are carried out and then integral pole placement control strategy is developed for the quarter car model and implemented for the closed loop control simulations. The effectiveness of this controller shows that the suspension travel is well controlled with given suspension travel overshoot within given settling time as compared to the passive quarter car model.*

1. INTRODUCTION

The automotive road vehicle manufacturers are continuously emphasizing in improving the suspension systems which can fulfil the essential objectives of the same. The important objectives of the suspension system in automotive road vehicles are to improve the ride quality of the vehicle by isolating the sprung mass; which is the mass of the vehicle which is suspended over the suspension system; from road disturbances, to keep good road holding, to support the vehicle's static weight by keeping the suspension rattle space (suspension travel/deflection) limits to minimal. The above mentioned objectives of the vehicle suspension system conflict with one another. For instance if a designer decides to develop a ride comfort tuned suspension then the developed suspension may not act nicely in road holding while cornering.

The suspension systems are broadly classified depending upon which suspension parameters are externally controlled as passive suspensions in which no parameters are not externally controlled, semi-active suspension system in which the spring rate and/or damping are modulated in response to the frequency of sprung mass motions and active suspension systems which incorporate the actuators to generate the desired forces in the suspension system [1].

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For improving the overall performance of automotive vehicles in recent years, suspensions incorporating active components have been developed. Passive suspension systems in automotive vehicles which include conventional components such as spring and damper which are time invariant [1]. This time invariant nature of the system makes the passive suspension system more rigid when the vehicle is excited by the variety of sources or different road surfaces. Active suspensions on the other hand incorporate the actuators to generate desired forces in the suspension system.

There is an enormous amount of work had been published in the fields of active suspension system design and development of the vehicle; and optimization of the suspension system of the vehicle. Some of these work pertaining to the present paper are summarized as follows.

One of the comprehensive piece of work in the field of multi objective optimization of the suspension system was reported by Gobbi et al. [2] in 2006. A two degree of freedom linear quarter car model was used along with randomly profiled roads. The performance indices considered for optimizing the suspension system were ride discomfort, road holding and suspension rattle space. The design variables chosen to be optimized were suspension spring stiffness, damping co efficient of the damper and the controller gain.

Akar et al. [3] in 2007 presented an integrated active suspension controller for vertical dynamics emulation. The proposed controller consisted of an active body controller and a force controller, that are both designed based on mathematical models derived from physical principles and also validated by experimental data.

Darus and Enzai [4] in 2010 investigated the performance of an active suspension of a quarter car model using Linear Quadratic Regulator (LQR) control and Proportional Derivative Integral (PID) control. It can capture basic performances of vehicle suspension such as body displacement, body acceleration, wheel displacement, wheel deflection, and suspension travels. LQR and PID control strategy performance observed depend on the changes of the road surface. Simulation was based on the mathematical model by using MA TLAB/SIMULINK software. Results showed that performance of body displacement and wheel displacement can be improved by using LQR and PID control scheme.

Ismail et al. [5] in 2012 applied the composite nonlinear feedback (CNF) to design the controller based on control law for an active suspension system. The CNF control law was composed of a linear control law and a nonlinear feedback part. The active suspension system was hierarchically divided into two subsystems with the system transformation based on the system structural properties at first and then the CNF control laws are designed for the two subsystems each One of the CNF control laws is designed for tracking control and the other is designed against the mismatching disturbance. The complete control law was made by combining the two CNF control laws and the system transformation. The simulation results showed that CNF controller is better than LQR controller and passive model.

The present paper investigates an active suspension system design using integral pole placement control for controlling the suspension rattle space, which is the maximum allowable relative displacement of the suspension components between the sprung mass and un-sprung mass of the vehicle. The active suspension system is physically modelled as a quarter car active suspension system using mass, spring, damper and actuators as two degrees of freedom system. The physical system is mathematically modelled first and presented in the section of quarter car dynamic model. The road profile adopted for the simulation is a step function. This dynamic system is then modelled in MATLAB with and without the proposed control scheme. The time versus rattle space simulations are obtained for quarter car passive and active systems and compared with each other in the simulation results section.

2. MATHEMATICAL MODELLING

To simplify the suspension analysis, quarter car vibration model with two degrees of freedom is used. Quarter car model consist of the 1/4th of the total mass of the vehicle. It can capture important characteristics of a full model. As seen in Fig. 1, the model consists of two different masses and two sets of spring and damper. Sprung mass, m_s, is considered only a quarter of the total mass supported above the suspension [6]. Therefore, the sprung mass includes, for example, car body, seats, internal components, and passengers. Un-sprung mass, m_u, is also considered a quarter of the total mass suspended below the suspension. The un-sprung mass then includes, for example, wheels, wheel bearings, brake rotors, and drive shaft. The vertical displacements of m_s and m_u are z_s andz_u respectively. The road height, which the wheel is contacted with is z_r. The suspension of the car locating between the sprung mass and the un-sprung mass is modelled as the spring coefficient, k_s , and the damping coefficient, c_s .The elastic and damping properties of the tire are modelled as the spring with its constant, k_t , and the damper with the coefficient,c_t. Note that the springs and dampers are characterized as linear system.

The quarter car model shown in Fig. 1 is passive, however, presented paper aims to design a control scheme for the active suspension system and hence the quarter car model with active suspension system components is utilized as shown in Fig. 2.

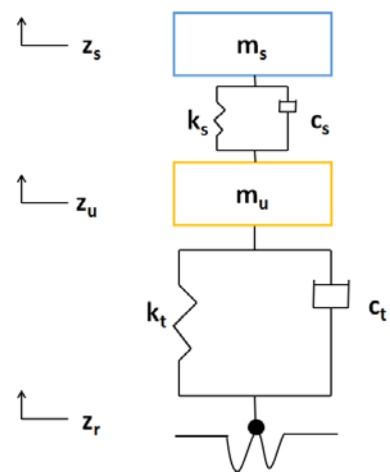

Fig. 1: Quarter car passive model

The quarter car active suspension model with low bandwidth type has an additional component between sprung and un-sprung masses shown by u. u is an active force actuator. Unlike passive suspension elements, the created force by this actuator does not depend directly on the relative displacement or velocity of the suspension, but also, it is determined by a sensory feedback controller working based on a special control strategy.The aim of the active system is to improve the suspension response just around the natural frequencies of the body of vehicles, with the typical range of 1 – 3 Hz. At higher frequencies, the actuator does not work effectively, and the wheel-hop motion is controlled only by the passive elements [7].

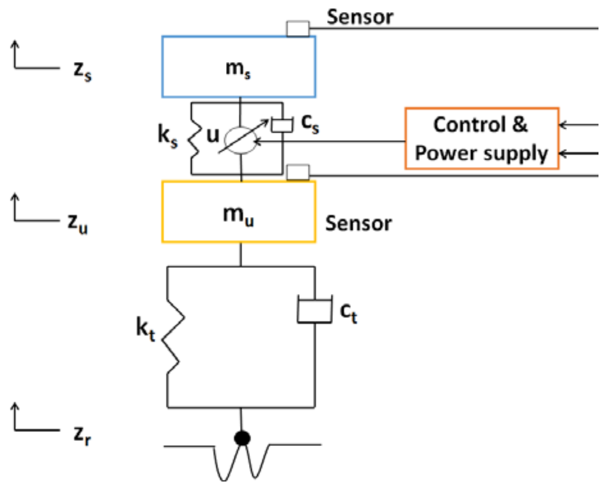

Fig. 2: Quarter car active model

The equations of motion of the system shown in Fig. 2 can be obtained by applying the Newton's second law of the motion to the system and can be given as:

$$m_s\ddot{z}_s = -k_s(z_s - z_u) - c_s(\dot{z}_s - \dot{z}_u) + u \quad (1)$$

$$m_u\ddot{z}_u = k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) - k_t(z_u - z_r) - c_t(\dot{z}_u - \dot{z}_r) - u \quad (2)$$

The state space variables for which can be given by:

$$X_1 = z_s \quad (3)$$

$$X_2 = \dot{z}_s \quad (4)$$

$$X_3 = z_s - z_u \quad (5)$$

$$X_4 = \dot{z}_s - \dot{z}_u \quad (6)$$

Here, $z_s - z_u$ is the suspension travel (rattle space), and$\dot{z}_s$ is the sprung mass velocity.

So the state space equations are given by:

$$\dot{X} = AX + Bu \quad (7)$$

$$Y = CX + Du \quad (8)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -{(c_s c_t)}/{m_s m_u} & 0 & ((c_s/m_s)*((c_s/m_s)+(c_s/m_u)+(c_t/m_u)))-(k_s/m_s) & -(c_s/m_s) \\ c_t/m_u & 0 & -((c_s/m_s)+(c_s/m_u)+(c_t/m_u)) & 1 \\ k_t/m_u & 0 & -((k_s/m_s)+(k_s/m_u)+(k_t/m_u)) & 0 \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} 0 & 0 \\ 1/m_s & (c_s*c_t)/(m_s*m_u) \\ 0 & -(c_t/m_u) \\ (1/m_s)+(1/m_u) & -(k_t/m_u) \end{bmatrix} \quad (10)$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \quad (11)$$

$$D = [0 \quad 0] \quad (12)$$

Table 1 shows the quarter car constants chosen for the simulations. The values listed are chosen from the Mathworks website tutorial [8].

Table 1: Quarter car constants

Symbol	Name	Values
m_s	Sprung mass	2500 Kg
m_u	Un-sprung mass	320 Kg
k_s	Suspension stiffness constant	80000 N/m
k_t	Tire stiffness constant	500000 N/m
c_s	Suspension damping co-efficient	350 Kg/sec
c_t	Tire damping co-efficient	15020 Kg/sec

3. CONTROL METHODOLOGY

Pole placement technique is a modern method for designing of control system.If it is assumed that the system is completely state controllable, its closed loop system poles can be placed at any desired location through state feedback by means of an appropriate state feedback gain matrix.

Open loop system can be defined by equation 7. X is a scalar vector and *u* is the control signal.The state X is not fed back to the control system *u*.

The results can be obtained by using transfer function in MATLAB.To this system applying linear state feedback control law of the form*u* = *-KX*. The K is a matrix called state feedback gain matrix.

Then, the closed loop system is given by the homogeneous equation [9]:

$$\dot{X} = (A - BK)X \quad (13)$$

This is a state feedback system as the system state X is fed back to control signal U.To achieve zero steady state error an integral action is required. Following is the augmented values of matrices which is obtained by adding integral state [8].

$$Aa = 1.0e{+}03 * \begin{bmatrix} 0 & 0.0010 & 0 & 0 & 0 \\ -0.0066 & 0 & -0.0253 & -0.0001 & 0 \\ 0.0469 & 0 & -0.0482 & 0.0010 & 0 \\ 1.5625 & 0 & -1.8445 & 0 & 0 \\ 0 & 0 & 0.0010 & 0 & 0 \end{bmatrix} \quad (14)$$

$$Ba = 1.0e+03 * \begin{bmatrix} 0 & 0 \\ 0 & 0.0066 \\ 0 & -0.0469 \\ 0 & -1.5625 \\ 0 & 0 \end{bmatrix} \quad (15)$$

$$Ca = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (16)$$

$$Da = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Feedback is used to change the behavior in a way which is more favorable for our specifications. The desired closed loop poles can be obtained by forcing them to be there by constructing a feedback control system and conducting a pole-placement design where a controller gain K is set.

For our specification, the required K is

$$K = 1.0e+05 * \begin{bmatrix} 2.3298 & 0.4500 & -1.4119 & 1.3706 & 0.9559 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

4. SIMULATION RESULTS

Open loop simulation results are shown in section 4.1 and closed loop simulation results in section 4.2

4.1 Open loop simulations

The Fig. 3 shows that change in $z_s - z_u$ (suspension rattle space/travel), when there is a step change in actuated force u (input 1).It can be seen from the Fig. 3 at 50 sec time, the suspension travel almost reaches the desired level.

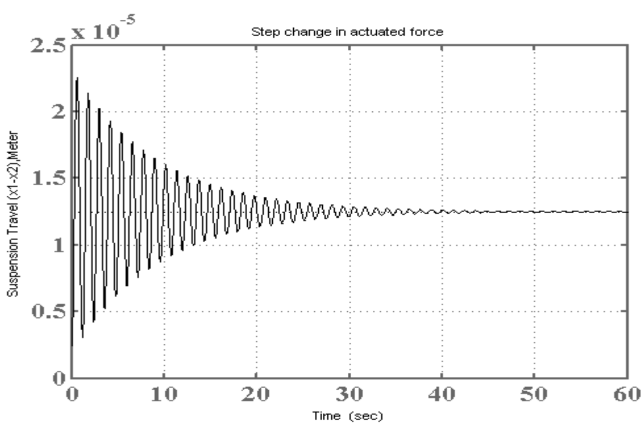

Fig. 3: Step change in actuated force

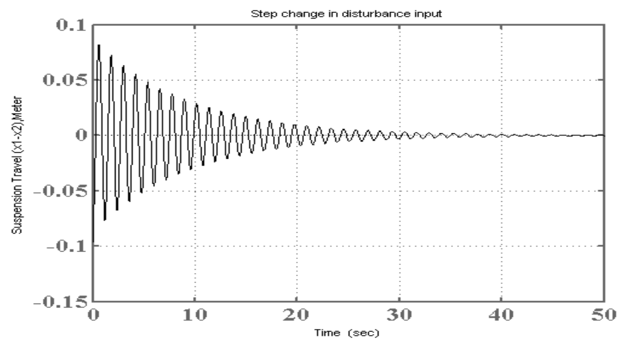

Fig. 4: Step change in disturbance input (z_r) with magnitude of 0.1 meter

Fig. 4 shows step change in the disturbance input (z_r) with magnitude of 0.1 meters.). As can be seen from the Fig. 4 at 50 sec time, the suspension travel almost reaches the desired level.

4.2 Closed loop simulations

A feedback controller should be designed in such a way that when the road disturbance (z_r) is simulated by a unit step input, the output ($z_s - z_u$) has a settling time less than 5 seconds and an overshoot less than 5%. The feedback controller as explained in section 3 has been modelled along with the quarter car state space equations in MATLAB script file which in turn calls the closed loop controller Simulink file for obtaining the desired output. The Simulink file used is shown in Fig. 5 below.

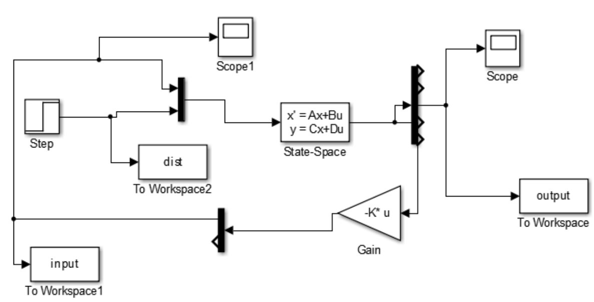

Fig. 5: Simulink closed loop controller file

Fig. 6 shows the two plots, the upper plot is showing the variation of the actuator force (u) with respect to time and the lower plot shows the external road disturbance (z_r) with respect to the time given to the system. z_r is a step input of 0.1 meters given at time 1 second.

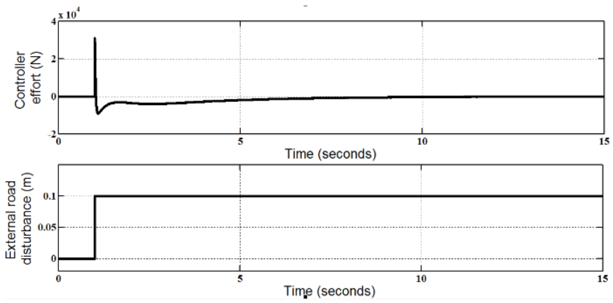

Fig. 6: Controller effort and Road disturbance

The MATLAB file for the closed loop controller was simulated for the quarter car constants (sprung and un-sprung masses, suspension's and tire's stiffness and damping co-efficient values,), these values are listed in table 1; along with the augmented matrices of the state space equation matrices (equations 7 to 12), damping ratio and natural frequency to find the state feedback gain matrix K value for the controller by Ackerman's formula. Damping ratio ξ is determined from $M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$. Here M_p is the fractional overshoot limit for our case it is taken to be 5% of open loop suspension travel. Moreover; natural frequency ω_n is determined from $t_s = {4}/{\xi\omega_n}$. Here, t_s is the settling time for our case it is taken to be 5 seconds.

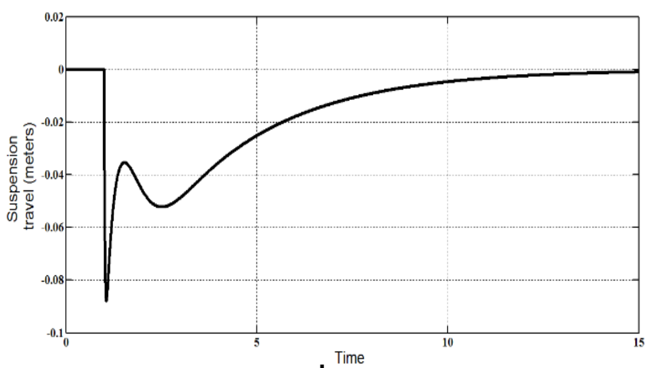

Fig. 7: Closed loop controller response

For these above mentioned values when the MATLAB script file for closed loop controller was simulated it gives the results as shown in Fig. 7. As can be seen from the Fig. that the suspension travel/rattle space reaches to the desired tolerance band of 5% in time less than 5 seconds.

5. CONCLUSIONS

The present paper has proposed an integral pole placement controller to control the suspension travel/rattle space to the desired limits set by the user, for our case these were chosen to be 5% is the fractional overshoot and 5 seconds is the settling time.

When the simulation results obtained from the open loop are compared with the ones obtained from the closed loops, it can be observed that the suspension travel is not reaching to the desired limits of suspension travel in desired time. Closed loop simulations show that, by using the proposed closed loop state feedback controller these limits can be reached.In this way, the proposed controller methodology, which is quite simple to implement when compared with the adaptive controller, gives the desired results of the suspension travel in desired time; and thus can work as an appealing controller scheme for the vehicles with low frequency active suspensions.

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